Examples to the lecture on types

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Example

$$
\begin{array}{llll}
D_1: & T & \rightarrow participants[Pers^+] & \text{2. single-label} \\
& Pers & \rightarrow person[Nam\ Id\ Dlic^?] & \text{3. local} \\
& Nam & \rightarrow name[\#] & \\
& Id & \rightarrow idCardNo[\#] & \\
& Dlic & \rightarrow drivLicNo[\#] & \# \in \mathcal{C}
\end{array}
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$$Nam \rightarrow name[\#]$$

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$$D_2 = D_1 \cup \{ U \rightarrow participants[Pers^{(7:100)}] \} \qquad \text{5. proper}$$

(hence 4. single-type)

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(hence 4. single-type)

$$D_3: \quad T \to participants[Driv\ Pers^{*}] \qquad \text{not 4. single-type}$$
$$Driv \to person[Nam\ Id\ Dlic]$$
$$Pers \to person[Nam\ Id\ Dlic^{?}]$$
$Nam, Id, Dlic$ like in D_1

Example (cnt'd)

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& Nam, Id, Dlic && \text{like in } D_1
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6. restrained-competition

(not 4., 5.)

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$Driv \rightarrow person[Nam\ Id\ Dlic]$ (not 4., 5.)

$Pers \rightarrow person[Nam\ Id\ Dlic^?]$

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$D_4:$ $\quad T \rightarrow participants[Pers^*\ Driv\ Pers^*]$ not 6.

the rest as in D_3

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the rest as in D_3

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$$D_5: \quad T \rightarrow participants[Pers^*\ Void\ Driv\ Pers^*]$$

$$Void \rightarrow nothing[\,]$$

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$Void \to nothing[\,]$

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Proper Type Definitions – Computing Intersection

Construct a type definition

for the intersection of types

defined by a proper D.

Express $[\![T]\!] \cap [\![U]\!]$ as $[\![TU]\!]$ where TU a new symbol

for each T, U such that $T \rightarrow l \ldots,\ U \rightarrow l \ldots \in D$.

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look at the labels (instead of whole terms, instead of type names).

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Intuition:

look at the labels (instead of whole terms, instead of type names).

Take T_0, U_0. The rules for T_0, U_0 in D

$T_0 \rightarrow l(\tau)$ $\qquad U_0 \rightarrow l(\upsilon)$ $\qquad$ where $(\,) = [\,]$ or $(\,) = \{\}$

$T_0 \to l(\tau)$ $\quad$ $U_0 \to l(\upsilon)$ $\quad$ where $(\,) = [\,]$ or $(\,) = \{\}$

$l(t_1, \ldots, t_n) \in [\![T_0]\!] \cap [\![U_0]\!]$ iff

$\exists_{T_1,\ldots,T_n,U_1,\ldots,U_n} \quad T_1 \cdots T_n \in L(\tau), \; U_1 \cdots U_n \in L(\upsilon), \; \forall_i \, t_i \in [\![T_i]\!] \cap [\![U_i]\!].$

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New symbol TU, for each T, U such that $T \to l\ldots$, $U \to l\ldots \in D$.

For each T in τ at most one such U in υ.
U in υ $\qquad$ T in τ

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So assume: ... exactly one ...

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The rule for T_0U_0 is $\qquad T_0U_0 \to l(\tau')$.